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as intimately as Egypt and possibly India. Similarly trigonometry did not have its initial development in that city. A serious fault in this work seems to the reviewer to be the neglect to emphasize the decimal character of our number system and the import of this fact for the instruction in arithmetic. Constant emphasis on the decimal character of our number system, and use of it, greatly simplifies the work of computation and leads to a better understanding of the processes of reckoning. For instance, on page 50 it is suggested that in mentally adding, 85, 25, and 40, we should first add 15 to 85 making 100, then add the remaining 10 of 25, and finally add 40. This, it seems, is contrary to all well known methods of rapid mental computation which universally proceed from left to right. In this problem the child should be taught to first add 20 to 80, then add 40, and finally add the two remaining fives. These suggestions may seem to be trifles but it is precisely in these trifles that pupils obtain or lose comprehension of the processes of arithmetic.

Both authors recommend the so-called Austrian, or additive, method of subtraction but neither one presents the application of this system to problems in long division in which, instead of writing down the partial products, one writes only the successive *remainders*. Thus to divide 13650 by 243 the plan is as follows:

$$\begin{array}{r} 243)13650(56 \\ 150 \\ 42 \end{array}$$

$5 \times 3 = 15$; $5 + 0 = 5$. $5 \times 4 = 20$; $20 + 1$ (carried) $= 21$; $1 + 5 = 6$. $5 \times 2 = 10$; $10 + 2$ (carried) $= 12$; $12 + 1 = 13$. The 0, 5, 1 are set down in turn and form the remainder when 5×243 is subtracted from 1365. Proceeding in the same way the next product 6×243 is not set down but simply the remainder. In most European schools where the additive method is taught this form of long division is suggested. It is easily mastered and saves much of the *longness* of long division.

Both of these works are notable and desirable additions to the literature on instruction in arithmetic. Mathematicians have somewhat neglected this field and, partly in consequence, the erroneous notion has become rather widespread that the less an individual knows about mathematics the better qualified he is to write on arithmetic and instruction in arithmetic. These works present the material from the standpoint of the trained mathematician who is at the same time familiar with the elementary schools. Their influence will be felt in elementary instruction.

L. C. KARPINSKI.

Analytic Geometry and Principles of Algebra. By ALEXANDER ZIWET, Professor of Mathematics at the University of Michigan, and LOUIS ALLEN HOPKINS, Instructor in Mathematics at the University of Michigan. New York, The Macmillan Company, 1913. In the series edited by E. R. Hedrick. viii+369 pages and 150 figures.

As far back as the oldest inhabitant can remember it has been considered the proper thing to write text-books, and publishers are usually eager to assist the unselfish missionary in presenting his views as to the proper manner of educating the masses. All too frequently the prospective author has responded to this high call before being kissed by the muse, and has been obliged to profit by the efforts of his predecessors to such an extent that each successive contribution secures an ever increasing degree of uniformity.

According to the historical researches of Heine, the massive wall was built about the city of Göttingen to prevent some enterprising docent from bringing new ideas into the university. To a large extent, so far as text-book writing is concerned, a very efficient wall of conservatism had been erected.

And then came a reaction; side by side with this conservative element a new faction was developed which represented the other extreme. We have algebras for boys, algebras for girls; geometry for engineers, geometry for medical students, and so on; probably we may soon expect trigonometry for red-headed students and even further differentiation.

Finally, once in a while we see the announcement of a text-book written by really chosen authors—persons of broad scholarship and ripe experience who undertake the task with a clearly defined purpose, having no axe to grind and fearless of charges of being too old or too new.

Since the book under review belongs distinctly to the third category, it demands more than a passing notice. The mechanical make-up of the book merits favorable comment; the large type, spaced lines, consistent use of italics, bold-face etc., the well executed figures, the full index, and the complete answers not only make for easy reading but put a big premium on neatness and orderliness on the part of the reader.

As the title indicates, the volume is concerned not only with analytic geometry, but also with the principles of algebra. Elementary algebra, plane and solid geometry and some knowledge of trigonometry are presupposed, but nothing of graphical processes nor of the subjects of advanced algebra.

The text begins much as the ordinary text on analytics, but the persistent use of $\Delta x = x_2 - x_1$ and the early introduction of determinants are innovations. Projection, polar coördinates, and vectors are all discussed before the equation of the straight line is mentioned. The projective forms of the equation are treated in great detail, that of one line being followed by that of a pencil, including the point and angle of intersection of two given lines. Here follows a second, more extensive development of determinants, including co-factors, minors, and elimination. Finally the normal form of the equation of a line is derived from the polar equation, and the usual applications made. Then follows a whole chapter on permutations and combinations, mathematical induction, and a systematic treatment of determinants, including multiplication.

In the chapter on the circle the ordinary treatment is enlivened by the use of determinants, the discussion of quadratic equations, inversion, pole and polar, pencils of circles and parametric representation. Since the quadratic equation may involve imaginaries, a full discussion of numbers, rational, irrational, fractional, complex, together with a geometric interpretation of each case is given. This chapter closes with a proof of DeMoivre's theorem. A well written chapter on the properties and graphical representation of higher polynomials includes the concepts of the first and second derivatives, maxima and minima, points of inflexion, roots of numerical equations, both rational and irrational, Horner's method, and the expansion of a polynomial by Taylor's series. It comprises everything usually provided in the theory of equations, yet comes in so naturally that it seems a necessary part of analytic geometry.

The chapter on the parabola adds to the topics of the traditional course the parametric representation, area of a segment, and a full derivation of Simpson's rule for the approximate area under any curve, also an application to shearing force and bending moment.

The two chapters on central conics are more conservative; they are first discussed from the definition of the locus of a point the sum (or difference) of whose distances from two fixed points is a constant, and later by means of the eccentricity, in which it is shown that all conics can be cut from a right circular cone.

Then follows a chapter on algebraic curves, including the determination of the number of constants, the equations of tangents and normals, double points at the origin, and a detailed study of a number of particular algebraic and transcendental curves, incidentally developing the theory of logarithms. The principles of this chapter are applied to the study of empirical equations and a number of practical suggestions are added regarding cross-section paper, logarithmic paper, etc.

The part on solid geometry is very brief. The positive directions of the coördinate axes are so chosen that the xy-plane as viewed from the positive z-axis is the same as the xy-plane as previously studied. Linear systems of planes are treated fully, including the derivation of the volume of the tetrahedron determined by four non-coplanar points. In the discussion of the sphere we find inversion, tangent cone, polar theory and linear systems touched upon; the other quadric surfaces are but briefly treated, yet ruled surfaces are considered, and contour lines mentioned. Rotation of axes is all put in small type; it includes the derivation of Euler's formulas. A short appendix illustrates abridged multiplication and division.

Within the compass of this volume are found most of the subjects understood under the heading of a first course in analytic geometry, an almost complete treatment of advanced algebra, and a wealth of other material not found in either. And the book nowhere seems to be overloaded, but every page is lively and attractive.

The changes from one subject to another are not digressions, but are made consistently with the sound pedagogic principle that each idea is developed as needed, whether the idea comes from the box labeled algebra, or calculus, or what not.

A reviewer is always supposed to find some fault, just to show how much wiser he is than the authors of the book under review, but in this case the reviewer feels a certain embarrassment, and even detects symptoms of a wish to have been the author himself. However, a few points may be mentioned. In discussing the bisectors of the angles between lines or between planes, the only method derived is valueless if one of the boundaries of the angle passes through the origin. Incidentally, does any book treat this subject properly? The whole discussion of positive and negative regions, so well done for curves, is hardly convincing when applied to lines and to planes. A considerable number of the later proofs are not complete, and in some cases rather too brief to be of great service, yet with proper skill on the part of the instructor may have influence in the right direction.

In those institutions in which advanced algebra is required for entrance, the book would either have to be abbreviated, or a number of topics repeated. Perhaps it is an argument to allow more alternatives for entrance, and make frank provision for teaching algebra to all in college.

VIRGIL SNYDER.

PROBLEMS AND SOLUTIONS.

B. F. FINKEL, CHAIRMAN OF THE COMMITTEE.

PROBLEMS FOR SOLUTION.

Special Notice.—Please reread the requests as to form of solutions on pp. 258–259 of the October 1913 issue. Unless these directions are observed by contributors, solutions must either be entirely rewritten by the committee or else rejected. Put all drawings on separate sheets.

MANAGING EDITOR.

ALGEBRA.

When this issue was made up no solutions had been received for numbers 401, 402, 404, and 406. Please give attention to these.

405. Proposed by E. J. MOULTON, Northwestern University.

Given the alternating series

$$s = 1 - 1/2 + 1/3 - 1/4 + 1/5 - \cdots.$$

(*a*) Let s_n be the sum of the first n terms of the series. Show that in order to make the difference $s - s_n$ numerically less than $1/2k$ (k a positive integer) it is necessary and sufficient to take $n = k$; hence s_{500} differs from s by less than .001. (*b*) Let s_n' be the sum of s_n and 1/2 the $(n + 1)$th term of the series. Show that the difference $s - s_n'$ is numerically less than $1/2n(n + 1)$; hence s_{22}' differs from s by less than .001.

406. Proposed by S. A. COREY, Hiteman, Iowa.

Solve the system of equations:

$$\begin{aligned}(1 - x)(a_1 + a_2y + a_3z) &= d,\\ (1 - y)(b_1 + b_2x + b_3z) &= g,\\ (1 - z)(c_1 + c_2x + c_{3y}) &= h.\end{aligned}$$

407. Proposed by E. B. ESCOTT, University of Michigan.

In computing the values of the natural logarithms of 2, 3, and 5 by the following formulas:

$$\begin{aligned}\log 2 &= 2(7P + 5Q + 3R),\\ \log 3 &= 2(11P + 8Q + 5R),\\ \log 5 &= 2(16P + 12Q + 7R),\end{aligned}$$